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PROBABILISTIC APPROACH TO DETERMINE PUMP OPERATING POINT

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SUMMARY

Pump operating point is defined as the intersection of system resistance curve and pump's head–discharge characteristics curve. In a given pumping system, there would be variation in system and pump characteristics. System resistance is impacted by changes in static head and flow resistance offered by various constituent parts. Pump characteristics is influenced by speed of rotation, dimensional variation, besides part to part deviations in case of mass–produced pumps.

The usual practice to analyse the pump operating point is to determine the maximum and minimum discharge and head possible, based on the intersecting extremes of pump and system characteristics. Then the maximum and minimum power inputs are deduced and the prime mover size fixed. This is a typical deterministic approach.

In probabilistic approach, the possible variations in pump and system characteristics are considered. Then transfer functions or equations for pump operating point – discharge vs head and discharge vs power input – are evolved and a statistical or Monte Carlo simulation performed. Output of such simulation would be a set of discharge and power input data. This output data could be further processed to yield probability or frequency distribution of pump discharge and input power, which would be useful in more accurately predicting energy consumption, prime mover size, range of operation and such.

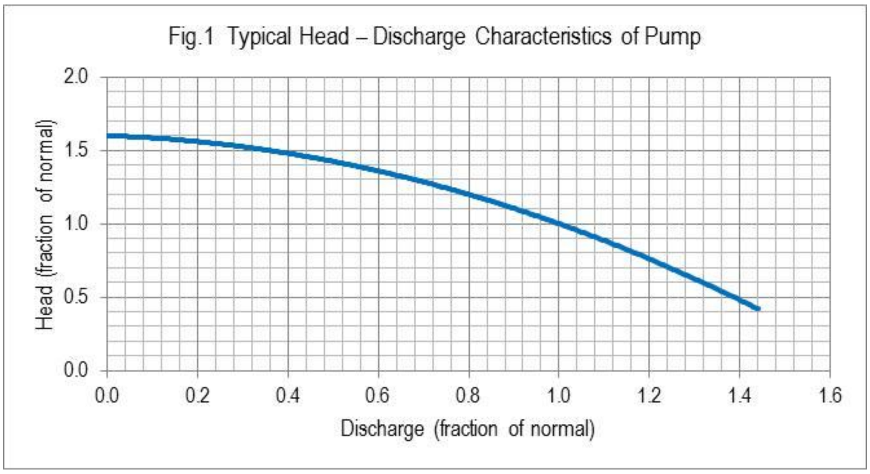
The same approach can be extended to other fluid handling systems including biological systems as well.

INTRODUCTION

In a pumping system, the point of operation where the demand of the system is met by the pump is defined as pump operating point. Determining pump operating point is important for arriving at accurate flow rate, head, power input and energy consumption. There are variations in both system characteristics and pump performance. Conventional deterministic approach is to estimate the nominal operating point and the extremities – maximum and minimum head and discharge possible. On the other hand in the probabilistic approach, the variation of the parameters such as flow rate, power input within the max and min possible range are predicted using statistical or Monte Carlo simulation. The obvious advantage of such probabilistic approach is that the behaviour of a parameter within the range is available; such information is not possible in conventional deterministic approach.

PUMP HEAD – DISCHARGE CHARACTERISTICS

Head – Discharge characteristics for a pump is the variation of head with discharge. In case of a rotodynamic pump, head decreases as discharge increases in most of the cases. However, in certain cases, the head may increase initially and then decrease as discharge increases from shut off or zero flow condition. Interpretations and reasons for such behaviour are dealt extensively in literature. Nevertheless, these pumps are operated at the drooping region of head discharge characteristics. Fig.1 shows head – discharge characteristics of a typical rotodynamic pump.



The head – discharge characteristics as seen in Fig.1 is nonlinear and can be represented by means of a polynomial in the form $y = a_0 + a_1x + a_2x^2$. Sometimes the complete characteristics may require a higher order polynomial for good representation. However, a segment of the characteristics can be successfully defined by a second order polynomial to a good degree of accuracy.

Thus, in the present discussions, the head discharge characteristics is represented by

$$H = a_0 + a_1Q + a_2Q^2 \qquad (1)$$

where Q is the discharge, H is the head and a_0, a_1, a_2 are constants.

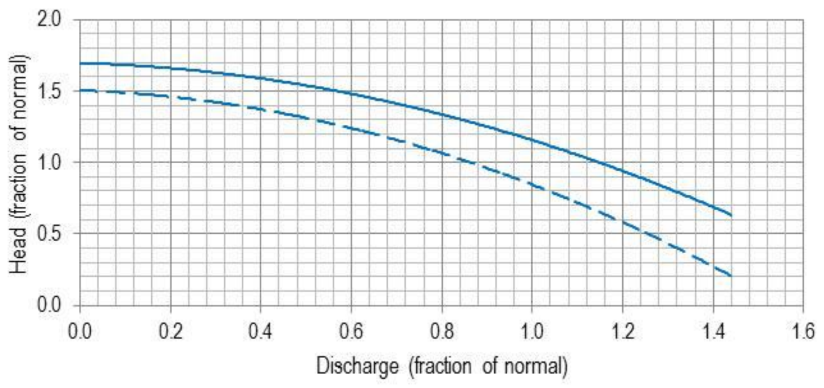
Factors impacting pump head – discharge characteristics

For a given pump model the H–Q characteristics are impacted mainly by speed of rotation and impeller geometry.

The effect of variation in speed of a rotodynamic pump is well known. Discharge varies proportional to the speed change and the head varies as square of speed change. Speed variation occurs due to change in the speed of prime mover. For example, if the prime mover is an electric induction motor, most common prime mover for rotodynamic pumps, any change in the line frequency would impact speed of rotation of the motor and the pump.

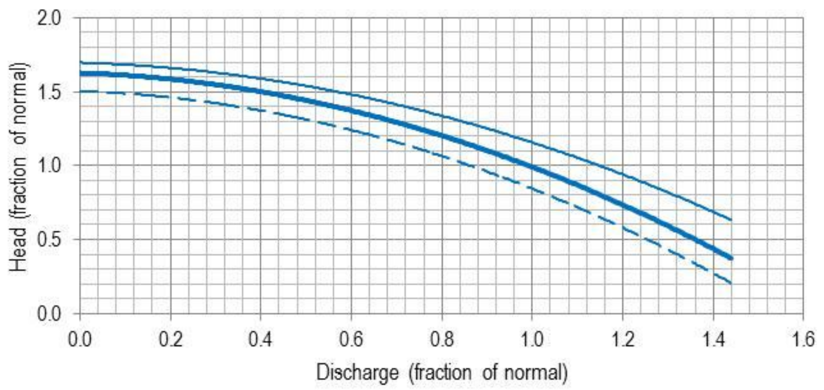
Major factors impacting pump characteristics under impeller geometry are outlet diameter, vane height, vane angles. These arise due to manufacturing variations. For example, in cast impellers, the geometry variation in part to part could be due to changes or degradation in patterns/moulds/dies.
These variations would impact the H–Q characteristics of the pump such that there would be upper and lower characteristic curves as shown in Fig.2.

Fig.2 Upper and Lower Bounds for Typical Pump H – Q Characteristics



In other words, for a given Q, there would be a max and min possible H. This transforms to coefficients a_0, a_1, a_2 used in Equation (1) having respective max and min values. This can be experimentally determined by testing a set of pumps a number of times to be statistically significant.
In a nutshell, H–Q characteristics would be a band within which the pump would operate. The operation of the pump can be simulated by assigning different values to the coefficients a_0, a_1, a_2 within their respective ranges. Fig.3 shows H–Q characteristics curve band and a typical characteristics curve lying within.

Fig.3 Pump H – Q Characteristics with Upper and Lower Bounds

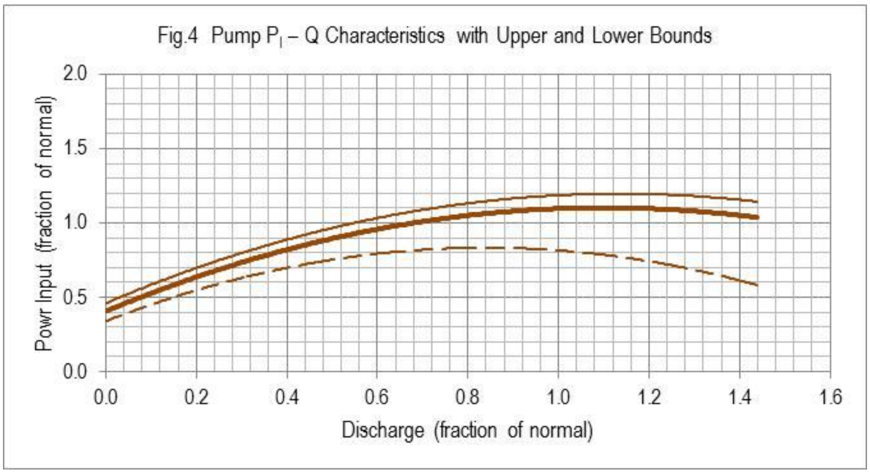


POWER INPUT CHARACTERISTICS
Power Input (P_I) characteristics of a rotodynamic pump is the variation of power input to the pump with discharge. Similar to H–Q characteristics, power input characteristics is non-linear and can be represented by a polynomial, usually third order.

$$P_I = p_0 + p_1 Q + p_2 Q^2 + p_3 Q^3 \quad \text{......(2)}$$

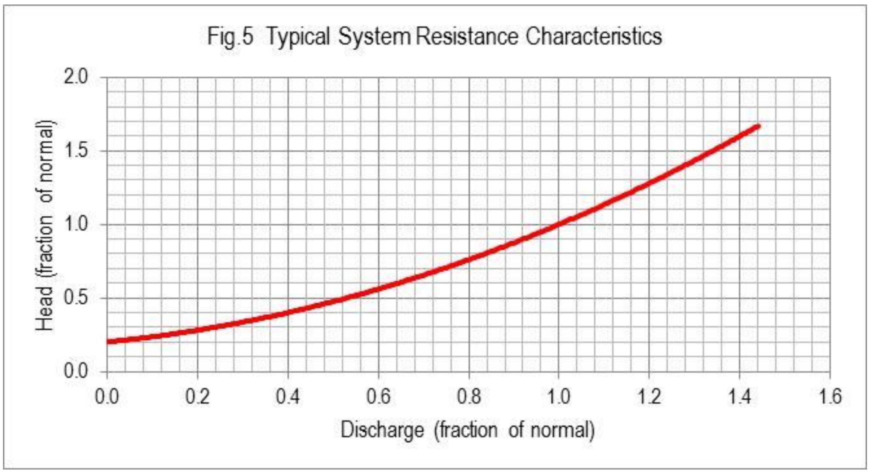
where P_I is the power input to pump, Q is the discharge and p_0, p_1, p_2, p_3 are coefficients of the polynomial.
For a pump model, these coefficients can be determined by testing a statistically significant set of sample pumps as in obtaining H–Q characteristics. The P_I characteristics also would be a

band, the coefficients having upper and lower bounds. An individual pump operation can be simulated by assigning values within bounds for the coefficients. Fig.4 shows the P_I characteristics band and a typical P_I characteristics curve within upper and lower bounds.



SYSYTEM RESISTANCE CHARACTERISTICS

System Resistance characteristics is the variation of system head with flow rate in a pumping system. System head is the head loss for a certain discharge to pass through the system. This is same as the energy to be imparted by the pump for certain discharge to take place through the system. This subject has been treated extensively in the literature. System head consists of static and dynamic portions. Static head is elevation through which the liquid need to be raised and does not vary with flow. Dynamic head is the resistance offered to the flow by the system – pipes, bends, valves, fittings and such. Dynamic head varies as square of flow. Fig.5 shows a typical system resistance characteristics.



System resistance characteristics is nonlinear and can be represented by a second order polynomial of the type $y = b_0 + b_1x + b_2x^2$. Here the constant term represents static head whereas the first and second order terms represent dynamic head. Thus

$H_S = b_0$ and $H_D = b_1Q + b_2Q^2$

$H_L = H_S + H_D = b_0 + b_1Q + b_2Q^2$ ....(3)

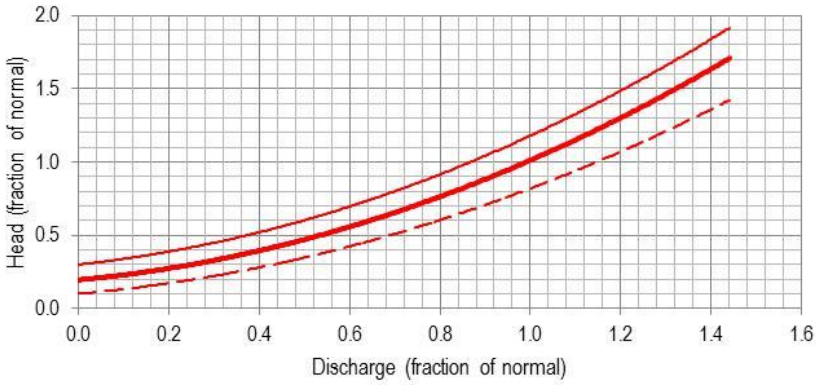
where H_S is the static head, H_D is the dynamic head, H_L is the system resistance head and Q is the discharge

Factors Influencing System Resistance Characteristics

In majority of the cases, system static head would be influenced by changes in the liquid level at the suction side. Examples are tidal movement in sea water pumping, level fluctuations in river

water intakes, accumulation and emptying in well water pumping. Besides, pressure at suction and discharge sides also influence system static head, common in process industries.
System dynamic head is the result of friction offered to the flow and so impacted by aging, scale/rust formation, sediment deposition, wear & tear, replacements, alterations and such changes in the system.
The possible variation of a typical system resistance characteristics can be obtained by analysing static head fluctuations, various elements involved in the system, and deducing maximum/minimum fluid friction. Then, maximum and minimum values of the individual coefficients in Equation (3). can be evaluated. Subsequently, similar to pump characteristics, a band having upper and lower bounds of system resistance characteristics can be arrived at. Fig.6 shows a typical system resistance characteristics contained within the upper and lower bounds.

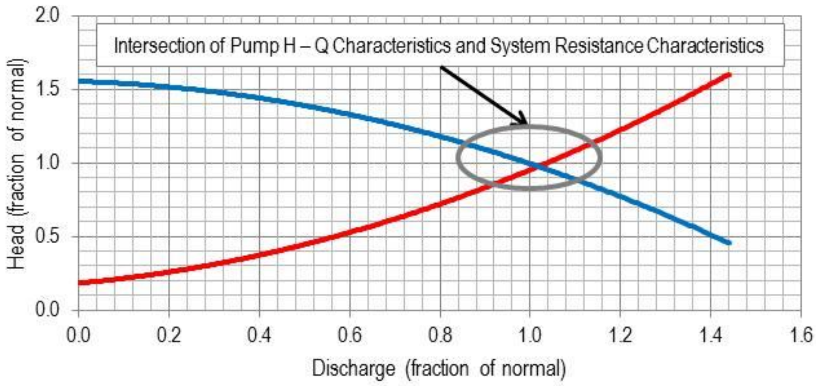
Fig.6 System Resistance Characteristics with Upper and Lower Bounds



PUMP OPERATING POINT

Pump operating point can be defined as the unique flow rate and head combination at which the head demand by the system is met by the pump. This is the intersection of pump H–Q characteristics curve and system resistance curve as shown in Fig.7.

Fig.7 Pump Operating Point



From Equation (1) and Equation (3), pump operating point would be

$$a_0 + a_1 Q + a_2 Q^2 = b_0 + b_1 Q + b_2 Q^2$$

This can be simplified as

$$(a_0 - b_0) + (a_1 - b_1)Q + (a_2 - b_2)Q^2 = 0$$

Solving for Q,

$$Q = \frac{-(a_1-b_1) \pm \sqrt{(a_1-b_1)^2-4(a_2-b_2)(a_0-b_0)}}{2(a_2-b_2)} \quad \ldots\ldots(4)$$

Substituting the solved value of Q in either Equation (1) or Equation (3), H can be obtained. Thus pump operating point (Q,H) can be determined when coefficients $a_0, a_1, a_2, b_0, b_1, b_2$ are available.

Furthermore, pump H–Q characteristics and system resistance characteristics could be of higher order polynomials, in a generic form as shown below.

$$H = a_0 + a_1Q + a_2Q^2 + \cdots + a_nQ^n$$

$$H_L = b_0 + b_1Q + b_2Q^2 + \cdots + b_nQ^n$$

These equations can be solved using techniques like Newton – Raphson method. In the present discussions, however, second order polynomials are considered for ease in demonstration/ presentation and because the characteristics curve segments in the operating range can be approximated to second degree polynomials with good accuracy.

MONTE CARLO SIMULATION

Monte Carlo simulation technique was developed during 'Manhattan Project', evolution of atomic pile. Today, it is being used in various fields such as engineering, economics, finance, physics, chemistry to name a few. Consequently, application of this technique differs among the fields/areas. Monte Carlo simulation is a stochastic method wherein system level predictions are made using the randomness and probabilities of individual elements impacting that system.

Monte Carlo simulation technique can be applied to determine pump operating point, the present discussions. Let $Y = f(X_1, X_2, X_3, \ldots)$, then the conventional method is to solve the equation for known values of $X_1, X_2, X_3, \ldots$; this is deterministic approach. This method would serve the purpose if the number of independent variables $X_1, X_2, X_3, \ldots$ and the variation within each of them is limited. However, when it is necessary to deal with large number of independent variables, variations within each of the variables are significant, interactions among the variables are considerable and combination of these factors are present, using conventional deterministic methods would be impractical. In such cases, Monte Carlo simulation comes in handy. Here the equation is solved by assigning randomly selected values to each X based on the probability of occurrence and the bounds – minimum and maximum values possible. The number of such iterations, called trials, depends upon available resources. The result would be several values of Y or a scenario of Y. More the trials, better would be the output scenario. To sum up, knowing the probability of occurrence or statistical distribution of each independent variable $X_1, X_2, X_3, \ldots$, statistical distribution of dependent variable Y can be obtained as depicted pictorially in Fig.8.

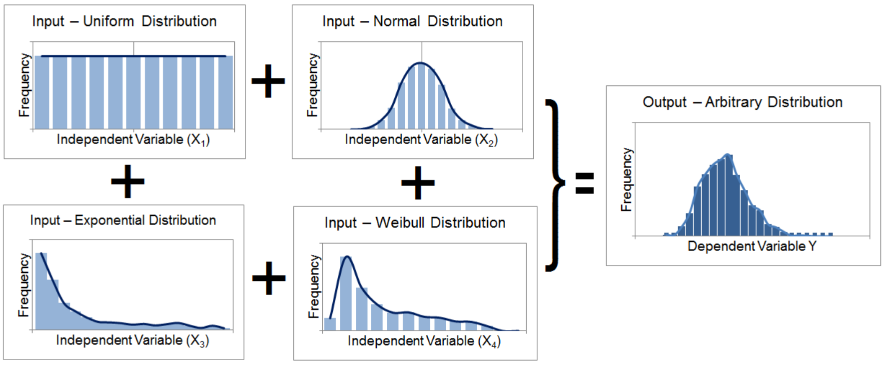


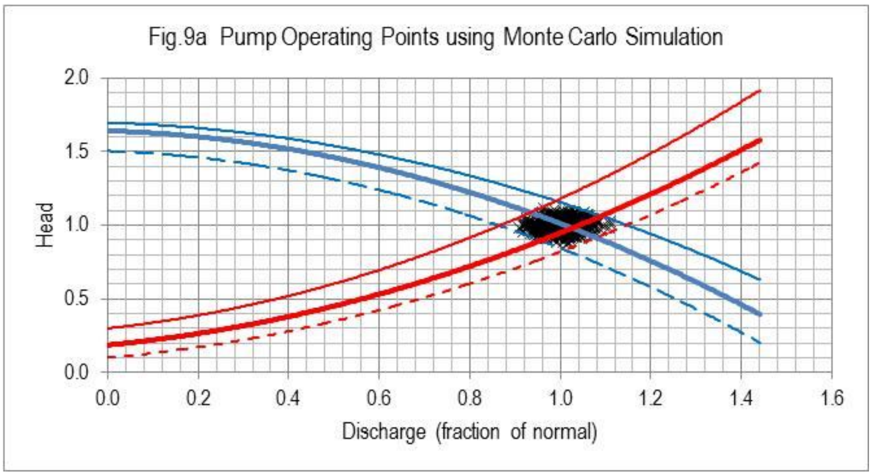
Fig. 8 Sample statistical distributions in $Y = f(X_1, X_2, X_3, \ldots)$

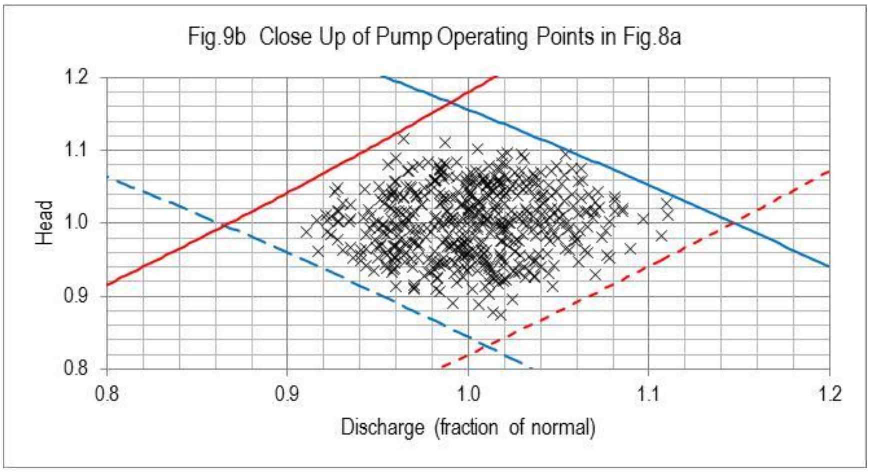
It has been shown earlier that pump H–Q characteristics and system resistance characteristics can be represented by polynomials, are bands having upper and lower bounds and any characteristics in between the bounds can be determined by assigning suitable values to the respective polynomial coefficients, and point of operation of pump can be obtained by solving the polynomials. Running Monte Carlo simulation for the current situation would involve

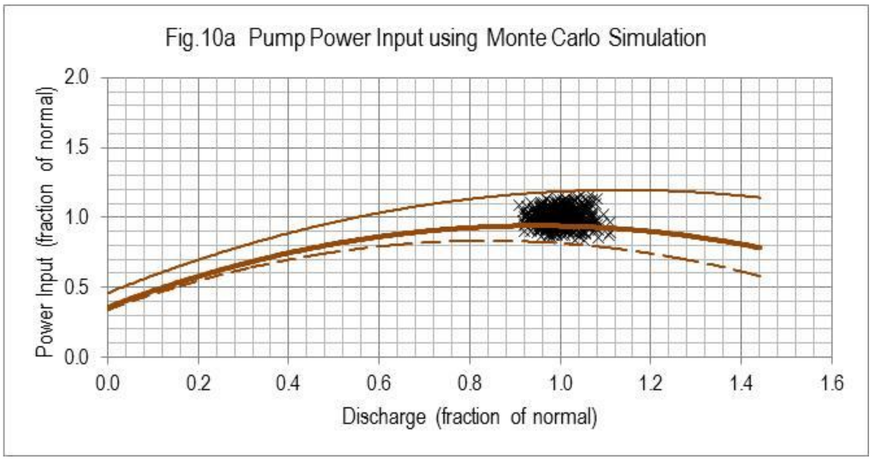
randomizing the polynomial coefficients or the independent variables for a certain number of trials and obtaining points of operation, the scenario of dependent variable. If from prior testing and estimation, the statistical distributions of these polynomial coefficients are known, that information can be used in Monte Carlo simulation for prediction of operating points to be more relevant. For simplicity, the present discussion considers uniform distribution for all the polynomial coefficients.
Following Table shows the assumptions made for the polynomial coefficients in the Monte Carlo simulation.

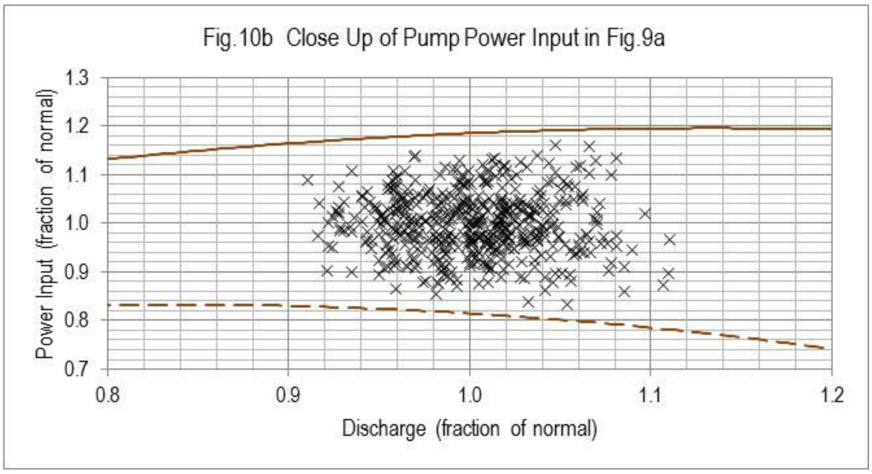
Coefficient	Maximum value	Minimum value	Type of distribution
a_0	1.7	1.5	Uniform
a_1	-0.09	-0.11	Uniform
a_2	-0.45	-0.55	Uniform
b_0	0.3	0.1	Uniform
b_1	0.33	0.27	Uniform
b_2	0.55	0.45	Uniform
p_0	0.46	0.34	Uniform
p_1	1.30	1.17	Uniform
p_2	-0.57	-0.70	Uniform

These polynomial coefficients are selected in such a way that the respective curves are fraction of normal, meaning that the operating point of consideration is at unity.
Fig.9a & Fig.9b show the result of Monte Carlo simulation for solving Q per Equation (4); corresponding P_l per Equation (2) are shown in Fig.10a and Fig.10b respectively. This is an outcome of 500 trials run in MS Excel© with above assumptions for the polynomial coefficients.

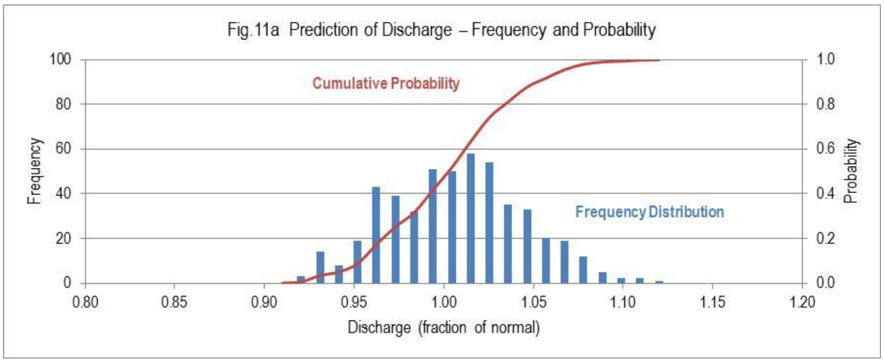


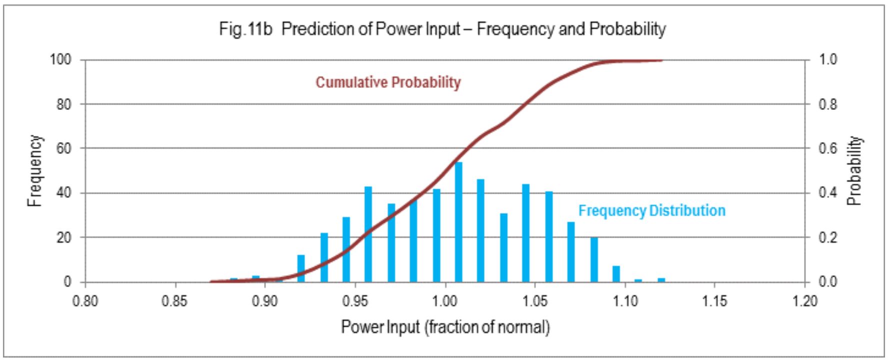






The solved Q and P_I values can be represented as frequency distribution as in Fig.11a and Fig.11b.





It can be observed from Fig9a, Fig.9b, Fig.10a and Fig.10b that the estimated operating points are randomly distributed within the bounds of pump and system characteristics. However, it can be noted from Fig.11a andFig.11b that the frequency and cumulative probability distributions for Q and P_I resemble normal distribution, though the polynomial coefficients were assumed to follow uniform distribution. Such data and the pictorial representation provide many useful information including maximum, minimum, most likely values for Q and P_I, besides other important statistical parameters. Further, these data and information can be used in design of the pumping system.
In the present example, conventional way to study the pump discharge would be compute minimum, nominal and maximum values, which work out to 0.865, 1.0, 1.147 respectively. Suppose the downstream application requires a minimum flow of 0.9, with Monte Carlo simulation results, it can be deduced that probability of achieving the required minimum flow is about 0.995 (99.5%); if the minimum required flow is 0.95, then probability of achieving it would be approximately 0.9 (90%). Such information would be useful in designing and analysing the pumping system. Possibly, having 99.5% probability (or 90% probability) in achieving minimum required flow is good enough compared to the efforts and investments required for further improvements. On the other hand, conventional deterministic approach would not be able to provide similar information.
Similarly, computation of power input P_I yields 0.77 and 1.2 as minimum and maximum values respectively. However, from the simulation results it can be inferred that when the overload capacity of the prime mover is 10% or 1.1 times the nominal, probability of power input to pump exceeding this limit is about 0.05 (5%) and if the overload capacity is increased to 15%, then probability of breaching the limit drops to 0.01 (1%). Such quantitative information would be useful in deciding about the type, sizing and overload capabilities of the prime mover.
Analogous to the above discussions on Q and P_I, variation of H can be analysed. As an extension to the above mentioned approach, energy input estimates can be arrived at by factoring in time parameter based on duration of operation.

CONCLUSIONS

The method presented and discussed here predicts the possible variation in a typical pump operating point. This method employs a probabilistic approach in analysing pump–system interaction, rather than conventional deterministic approach. The latter method typically establishes the nominal and extremities in the operating region whereas the former is better as it provides more useful information – besides the extremities, it shows the variation of Q or P_I within the extremities. This method would be useful in studying a pumping system, contemplating improvements, sizing prime mover and such. The above probabilistic method can be extended to multiple – parallel, series – pumps operation as well.
Further, similar probabilistic approach can be adapted to analyse pneumatic system involving compressor/s and biological systems concerning bio–fluids.

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